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A Constructivist Approach to Teaching

The development of a constructivist theory of knowing has been the focus of my interest for several decades. It was a philosophical interest that arose originally out of work concerning first the structure and semantics of several languages and later cognitive psychology. The title of this chapter, therefore, may need an explanation. Rosalind Driver, Reinders Duit, Heinrich Bauersfeld, and Paul Cobb, can speak about teaching from their own immediate experience, whereas I have never taught any of the subjects that you are experts in. So when I focus on the theory of constructivism, you may wonder why on earth a proponent of such a very peculiar theory of knowing should have anything to say about education in mathematics or science. It is a question I have often asked myself. If all goes well, you will see some justification at the end of my essay.

One very general observation has encouraged me to move in this direction. Education may never have been considered good enough, but whatever its methods and effectiveness were, it seems to have suffered a decline during the last twenty or thirty years. Today, there is a general consensus that something is very wrong, because children come out of school unable to read and write, unable to operate with numbers sufficiently well for their jobs, and with so little knowledge of the contemporary scientific view of the world that a large section of them still believe that the phases of the moon are caused by the shadow of the earth.

This has been said not only in official reports, but recently also by a particularly keen observer of society, the comedian Mark Russell. In one of his talks, he recently made exactly the three points that I just mentioned. The audience laughed, because what they expect from a comedian is parody or jokes. But in this case, he was being serious. Then he added: “Give the teachers more money, and they will teach the right answers.” That was the parody.

Unfortunately this remark portrays only too well an attitude that has gained ground through the years in school boards, commissions and also, of course, in Washington. It is a fatal attitude. Money does not change the philosophy of education. And a philosophy of education that believes in teaching right answers is not worth having.

As a constructivist, I cannot pretend to have an “objective” view of how this dismal misconception came about. But I have a view nevertheless. As I see it, the main root of the trouble is that for fifty years in this century we have suffered the virtually undisputed domination of a mindless behaviorism. The behaviorists succeeded in eliminating the distinction between training (for performance) and teaching that aims at the generation of understanding. All learning was reduced to a model that had been derived from experiments with captive pigeons and rats. Its fundamental principle was the “law of effect,” in which Thorndike (1898) had formulated the not altogether novel observation that animals tend to repeat the actions which in their experience led to satisfactory results. The behaviorists reformulated this by saying that any response that is “reinforced” will be repeated, and then they turned it into a “learning theory” based on the power of reinforcement.

For education, this learning theory has had unfortunate consequences. It has tended to focus attention on students’ performance rather than on the reasons that prompt them to respond or act in a particular way. Reinforcement fosters the repetition of what gets reinforced, regardless of the acting subject’s understanding of the problem that was posed and of the inherent logic that distinguishes solutions from inadequate responses. Training, thus, may modify behavioral responses but leaves the responding subject’s comprehension to fortunate accidents.

PROBLEM-SOLVING

Some fifteen years of research on reasoning at the University of Massachusetts have shown that first-year physics students come quite well trained to give the “right” answers to standard questions. However, when asked to solve a simple problem that is in some way different from the familiar ones of the textbooks, they reveal that they have no understanding whatever of the conceptual relationships indicated by the symbols in the formulas which they have learned by heart.

It certainly is not one single factor that is responsible for this state of affairs. I here want to suggest at least a couple. One is the still wide-spread notion that competence in intelligent behavior could be achieved by drilling performance. This belief has been thoroughly exploded. The many references in contemporary reports to the need to teach problem-solving are an eloquent symptom. The solving of problems that are not precisely those presented in the preceding course of instruction requires conceptual understanding, not only of certain abstract building blocks but also of a variety of relations that can be posited between them. Only the student who has built up such a conceptual repertoire has a chance of success when faced with novel problems. And concepts cannot simply be transferred from teachers to students – they have to be conceived.

The second factor is more delicate and perhaps more insidious. Science, having to a large extent replaced religion in the twentieth century, is all too often presented as the way to absolute truth. Yet even high school students have the intuitive awareness that the certainty of mathematical results is something different from the truth claims of biology or physics. If mathematics were explained as a way of operating with a particular kind of abstractions and science as a way of building models to help us manage the world we experience some of the latent resistances might be allayed. But this, again, would require some delving into conceptual foundations.

It is the growing awareness of this need for conceptual development that has begun to raise the question of how conceptual development should be approached and how it could possibly be fostered. These are questions about knowledge, questions that concern its structure as well as its acquisition. In order to answer them, one needs a theory of knowledge or, as philosophers say, an epistemology. This is the very area in which constructivism has attempted to introduce a new perspective.

Before explaining some aspects of the constructivist approach, I want to forestall a misunderstanding that I may have sometimes helped to create. From what I have said, it should be clear that I am interested in conceptual understanding, and in performance only insofar as it springs from, and thus demonstrates such understanding. What I am going to say will deal exclusively with the construction of conceptual knowledge. This does not mean that, from the constructivist point of view, memorization and rote learning are considered useless. There are, indeed, matters that can and perhaps must be learned in a purely mechanical way. The teaching of these matters, however, does not present problems beyond the problem of generating the required discipline in the students. Although I believe that a constructivist approach to conceptual development can help to engender a rapport between teacher and student and a propitious mood among the students, the creation of discipline is essentially a task with which teachers have far more experience than any theoretician.

CONSTRUCTIVISM

Although I will not continually cite him, I sincerely hope that you will realize at the end of my talk that almost everything I say today, can be said only because Piaget spent some sixty years establishing the basis for a dynamic constructivist theory of knowing.

The reviewer of a paper I recently wrote made a remark that truly delighted me. Constructivism, she said, is postepistemological.[1] I am sure you have all come across the now fashionable expression “post-modernist.” Post-epistemological not only fits this fashion, it also helps to convey the crucial fact that the constructivist theory of knowing breaks with the epistemological tradition in philosophy.

Constructivism arose for Piaget (as well as for Giambattista Vico, the pioneer of constructivism at the beginning of the 18th century), out of a profound dissatisfaction with the theories of knowledge in the tradition of Western philosophy. In this tradition, the basic epistemological concepts have not changed throughout the 2500 years of our history, and the paradox to which these concepts lead has never been resolved. In this tradition, knowledge should represent a “real” world that is thought of as “existing,” separate and independent of the knower; and this knowledge should be considered “true” only if it correctly reflects that independent world.

THE CONCEPT OF KNOWLEDGE

From the very beginning in the 5th century B.C., the sceptics have shown that it is logically impossible to establish the “truth” of any particular piece of knowledge. The necessary comparison of the piece of knowledge with the “reality” it is supposed to represent cannot be made, because the only rational access to that reality is through yet another act of knowing.

The sceptics have forever reiterated this argument to the embarrassment of all the philosophers who tried to get around the difficulty. Nevertheless, the sceptics did not question the traditional concept of knowing.

This is where constructivism, following the lead of the American pragmatists and a number of European thinkers at the turn of this century, breaks away from the tradition. It holds that there is something wrong with the old concept of knowledge and it proposes to change it rather than continuing the same hopeless struggle to to find a solution to the perennial paradox. The change consists in this: Give up the requirement that knowledge represent an independent world, and admit instead that knowledge represents something that is far more important to us, namely what we can do in our experiential world, the successful ways of dealing with the objects we call physical and the successful ways of thinking with abstract concepts.

Very often when I say this, there are some who protest that I am denying reality. It is foolish to deny the existence of reality, they say, it leads to solipsism, and solipsism is

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[1] N. Noddings in her introductory chapter to Constructivist views on the teaching and learning of mathematics, C.A. Maher, R.B. Davis, and N. Noddings (Eds.), *Monograph of the Journal for Research in Mathematics Education*, 1990.

unacceptable. This is a basic misunderstanding of constructivism, and it springs from the resistance or refusal to change the concept of knowing. I have never denied an “absolute” reality, I only claim, as the sceptics do, that we have no way of knowing it. And as constructivist, I go one step further: I claim that we can define the meaning of “to exist” only within the realm of our experiential world and not ontologically. When the word “existence” is applied to the world that is supposed to be independent of our experiencing (i.e. an “ontological” world), it loses its meaning and cannot make any sense.

Of course, even as constructivists, we can use the word “reality,” but it will be defined differently. It will be made up of the network of things and relationships that we rely on in our living and of which we believe that others rely on, too.

KNOWLEDGE IS ADAPTIVE

From the constructivist perspective, as Piaget stressed, knowing is an adaptive activity. This means that one should think of knowledge as a kind of compendium of concepts and actions that one has found to be successful, given the purposes one had in mind. This notion is analogous to the notion of adaptation in evolutionary biology, expanded to include, beyond the goal of survival, the goal of a coherent conceptual organization of the world as we experience it.

An animal that we call “adapted” has a sufficient repertoire of actions and states to cope with the difficulties presented by the environment it lives in. The human animal achieves this with relative ease; but the human thinker must also cope with the difficulties that arise on the conceptual level. The independent “reality” relative to which one speaks of adaptation does not become accessible to human cognition, no matter how well adapted the knower might be. This reality remains forever behind the points where action or conceptualization failed.

The shift to this “post-epistemological” way of thinking has multiple consequences. The most important is that the customary conception of “truth” as the correct representation of states or events of an external world, is replaced by the notion of viability.[2] To the biologist, a living organism is “viable” as long as it manages to survive in its environment. To the constructivist, concepts, models, theories, etc., are “viable,” if they prove adequate in the conexts in which they were created. Viability – quite unlike “truth” – is relative to a context of goals and purposes. But these goals and purposes are not limited to the concrete or material. In science, for instance, there is, beyond the goal of solving specific problems, the goal of constructing as coherent a model of the experiential world as possible.

NECESSARY CONCEPTUAL CHANGES

The introduction of the concept of viability does away with the notion that there will be only one ultimate Truth that describes the world. Any description is relative to the observer from whose experience it is derived. Consequently, there will always be more than one way of solving a problem or achieving a goal. This does not mean that different solutions must be considered equally desirable; however, if they achieve the desired goal, the preference for a particular way of doing this cannot be justified by its “rightness,” but only with reference to some other scale of values such as speed, economy, convention, or “elegance.”

[2] One might say, of course it is we who think of these stars as a W, but they do form a “real” group in the sky. But this is an illusion. Given the distances between them, each one of these stars has closer neighbors, and if one looks at Cassiopeia through a telescope, one can see quite a few of them inside and near the W, so that innumerable other groupings could be seen as a constellation.

These are conceptual changes that are difficult to carry through. And if one seriously adopts the constructivist approach, one discovers that many more of one's habitual ways of thinking have to be changed. But rather than burden you with further theoretically unsettling particulars, I will give you some experiential examples of conceptual building blocks that are our own construction.[3] It may help to make the constructivist view seem a little less unwarranted.

THE REALITY OF A CONSTELLATION

You all occasionally look at the sky at night and maybe you recognize some of the constellations. Among the constellations in the Northern hemisphere that were well known at the beginning of Greek culture in the first millennium before Christ, one is called Cassiopeia. If you know the Big Bear or Big Dipper, the Cassiopeia is opposite it, on the other side of the Polar Star. It has the shape of a W or, as the Greeks said, of a crown. The shape has been known and recognized for thousands of years, and it served the navigators of all times to find their way across the seas. It has not changed and it proved as reliable, as "real," as any visual percept can be.

For an astronomer, the five stars that are taken to compose the W have Greek letters as names, and the astronomer can tell you how far these stars are from us who observe them from our planet. Alpha is 45 light years away, Beta 150. The distance to Gamma is 96, to Delta 43, and to Epsilon 520 light years. – Let us consider this spatial arrangement for a moment. If you moved 45 light years towards Cassiopeia, you would have passed Delta and you would be standing on Alpha. The constellation would have fallen apart during your journey. If you moved sideways, it would disintegrate even more quickly. Where, then, does this image we call Cassiopeia "exist"? The only answer, I suggest, is that it exists in our minds. Not only because it is relative to the point from which we look, but also because it is we who pick five specific stars and create a connection between them that we consider appropriate.[4] This picking out and connecting is part of what I call the subjective construction of our experiential world.

THE COASTLINE OF THE BRITISH ISLES

A few years ago, a mathematician by the name of Benoit Mandelbrot invented what has become famous as the theory of fractals. In one of the presentations of his theory, he posed a question that seemed quite ridiculous. He asked, what is the length of the coastline of the British Isles? At first glance, there seems to be no problem at all. If the figure is not already known, one would simply make the necessary measurement. But here is the hidden question: How should one measure it? If you do it by the usual method of triangulation, what you measure are the distances between the points you choose for your triangulations, not the coastline. Clearly, if you took a foot ruler and actually measured the coastline, you would run

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3 Extensive analyses of the construction of conceptual fictions were compiled by Jeremy Bentham (ca.1780) and Hans Vaihinger (1913). It is important to realize that the word "fiction" does not indicate a negative evaluation but refers simply to conceptual structures that are applied to, rather than derived from experience. (Newton's laws, for instance, are at least partially based on the "fiction" of motion continuing to infinity unless some external force acts on the moving body.)

4 As my colleague Klaus Schultz remarked, it is characteristic of this conventional reality that, during the moment of terror, one may begin to hope that the opponent does not notice the checkmate possibility – but one never doubts that he or she could see it. It is indeed this intersubjectivity that makes conventional fictions so "real."

into difficulties. Apart from the time it would take, there would be innumerable places where you would have to decide whether the waterline round a rock, a sandbank, or a pebble should be counted as coastline or not. And imagine what would happen if you had to do the measuring on the level of molecules-it could not be done at all. In either case, the result would obviously be very much larger.

Again one might ask, where does the coastline of the British Isles "exist"? And again the answer has to be that it is something we construct, something that is very reasonable and appropriate in the conceptual contexts in which we want to use it. Take away the conceptual contexts we have created, and the notion of coastline ceases to have meaning.

THE IDEA OF EQUILATERAL TRIANGLE

The third example is a little nearer home for teachers. You go to a chalk board, draw something, and then you turn to your class and say, this is a triangle, and because its sides are the same length, we call it "equilateral." Those students in your class who happened to be listening, have no difficulty in understanding what you said. They could now all draw an equilateral triangle for themselves. That is not the problem. The point I want to make is that neither the triangle you drew on the chalk board, nor those the students are now drawing with the help of their rulers, are truly equilateral; and since they should consist of three continuous straight lines, they are not truly geometrical triangles. Precise measurement would reveal that their sides are not exactly equal, and magnification would show that their lines are loosely aligned successions of marks and therefore neither continuous nor straight.

Yet. you and the class know what you are talking about. You have in mind a structure that is made up of three perfectly straight lines whose length is exactly the same. Such a structure "exists" nowhere, except in heads. Yours, the students,' and anyone else's who knowingly uses the term "equilateral triangle."

This sounds very much like what Plato said about "perfect forms." But Plato was not a constructivist. Plato argued that such perfect abstract ideas originated with God, who instilled them into souls. And since the souls migrate from one incarnation to the next, we all have these ideas from the moment we are born. They are embedded in us from the start, although we do not know them until some quite imperfect experience calls them up. This is a beautiful theory. But for a constructivist who believes that explanations should, wherever possible, be rational rather than mystical or mythological, it is not a satisfactory one. From our point of view, to assume that something is God-given or innate, should be the very last resort, to be accepted only when all attempts at analysis have broken down.

In the case of triangles and other geometric forms, we can do much better. We can show that straightness and continuity are not abstracted from imperfect sensory impressions but from the movements of attention in the dynamic construction of images we create in our minds. They are, in fact, what Piaget called "operativ" rather then "figurative" or sensory structures, because they are abstracted from operations we ourselves carry out.

THE REALITY OF CONVENTIONAL RULES

A context in which one can experience the power of conceptual constructs with overwhelming intensity is the game of chess. I am sure all who have played that game are only too familiar with the powerful feeling of horror that grips you when you suddenly realize that, with the next move, your opponnent can put you into a checkmate position. Your heart begins to beat, your hands tremble, and the paralysis that grasps you is among the most unquestionably "real" experiences you can have. Yet, what is the cause of this physical ordeal? Where is the situation that horrifies you to such an extent? You cannot pin it to the chessmen or the board. It resides entirely in the rules and relationships that you have constructed in your mind and which you have somehow promised yourself – and your prospective opponents – to respect and maintain while you are playing the game of chess. You have decided to stick with those rules and to respect the agreed-on relationships because if you did not, you would no longer be playing the game of chess; and to play chess is what you decided to do.[5]

THE IMPORTANCE OF SOCIAL INTERACTION

Playing chess is a social activity and the way one acquires knowledge of the rules and conventions that govern the game is through social interaction, of which language is probably the most frequent form. This is obvious in games such as chess, but I would claim that social interaction is no less essential in the acquisition of the basic geometric forms and of a multitude of far more general concepts such as "coastline."

Much recent writing has stressed the social component in the development of conceptual knowledge and the term "social constructionism" has been used to distinguish this orientation from the "radical constructivism" some of us have been propagating. A little clarification would seem in order.

Piaget, who is undoubtedly the most important constructivist in this century, has been criticized, mainly on this side of the Atlantic, for not having considered social interaction in his theory of cognitive development. I think, this criticism is unjustified. If one reads Piaget's original works with the necessary attention – by no means an easy task, because his explanations are not always immediately transparent – one finds that somewhere in almost every book he reiterates that the most important occasions for accommodation arise in social interaction.

It is quite true that Piaget did not spend much time on working out the details of how social interaction is supposed to work. He was predominantly interested in something else, namely the logical structures by means of which the developing child organizes the world it experiences.

For Piaget, just as for the contemporary radical constructivists, the "others" with whom social interaction takes place, are part of the environment, no more but also no less than any of the relatively "permanent" objects the child constructs within the range of its lived experience. That is to say, it is the subject's interaction with constructs of its own that have proven viable and have been categorized as permanent external objects.

THE CONSTRUCTION OF "OTHERS"

If one takes this position, a question will sooner or later arise: How do these "others," the other people with whom the child populates its experiential world, come to be different from the innumerable physical objects the child constructs? The question focuses on a point that seems crucial for constructivism. If all knowledge is the knowing subject's own construction, how can one know of other subjects? I have tried to answer this question, by suggesting an hypothetical model in some of my papers.[6] The model is based on a passage in the first edition of Kant's

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5 See for example von Glasersfeld "Steps in the construction of 'others' and 'reality'" (1986) and "Facts and the self from a constructivist point of view" (1989).

6 This, of course, does not refer to the notion of "truth" and "necessity" within the rule-governed systems the experiencer constructs and decides to maintain. This is illustrated by the fact that within abstract, timeless systems, such as the syllogism or arithmetic, there is deductive certainty. But this

Critique of pure reason. We can only conceive of another subject, Kant wrote there, by imputing our own subjectness to another entity (1781, A354). In order to develop viable ways of acting in its experiential environment, the child learns to make predictions about the objects it constructs. The glass you hold will drop if you let go of it, and it will break when it hits the ground. The lizard you want to catch will dart away if it sees you. To learn this, you have to impute the capability of seeing to lizards. The entity you call "Daddy" will tell you not to do this or that because you might hurt yourself. To think this, you have to impute to Daddy (and to other entities like him) the capability of making predictions similar to, and perhaps even greater than, the predictive capability you yourself are using. In this way you construct "others" out of elements of yourself, and soon these others contribute to the image of your self.

This hypothetical model, clearly, would need a lot of elaboration to become a plausible model. For the moment, it will have to do as an example. I am using it only to show that if we do not want to assume some innate or mystical knowledge of the "existence" of other thinking subjects, we must find a way of explaining our knowledge of others on the basis of individual experience. That is to say, we must generate an explanation of how "others" and the "society" in which we find ourselves living can be conceptually constructed on the basis of our subjective experience.

THE CONSTRUCTION OF PLURALITIES

A last example that may be of interest to those who teach arithmetic to children. It came out of the work that Les Steffe, John Richards, Paul Cobb and I did for our book on Children's counting types (1983).

If you have read any text in the philosophy of mathematics will know that the intuitionists, starting with Brouwer and Heyting, differ from the formalists in their definition of number. The intuitionist, roughly speaking, hold that number arises when you create a unit, then a second unit, and then you join the two together to form a new conceptual unit. Brouwer calls this a "two-oneness." A somewhat awkward expression, but it does capture the most important characteristics of number, namely that it is a repeatable unit and recursively generates other units. And this can be repeated to infinity.

Here I am not concerned with infinity (which, of course, would be a very interesting subject). I am concerned with the very beginning of the construction. This beginning does not involve number words. There is merely the creation of two entities, and then a kind of "stepping back" and considering them together. This is the origin af an entity that contains more than one, i.e., a "plurality."

If we take this as a working hypothesis, we can now ask the question: how does it come about that normal children, sometime between the age of 14 and 24 months, learn to use plural words of their language? In order appropriately to use the plural form of, say "apple," you have to know that there is more than one apple on the table in front of you. You don't have to know how many, but you have to know that there is more than one. – How did your baby daughter, come to know this?

Here is the scenario I have developed. Your daughter must first of all have learned to isolate a particular kind of discrete, unitary item in her experiential field, and she must have associated that kind with the word "apple." Now she looks at the table and recognizes one of these items. She may, as children often do, label that item and utter the word "apple." Neither this apple she has singled out on the table, nor any of the others, provides a sensory

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certainty does not pertain to premises or units that are inductively inferred from the realm of temporal experience.

characteristic which indicates that there are more than one and that the plural “apples” would be appropriate. Plurality is not a sensory property.

Plurality is the conceptual construct of an observer, i.e., an experiencing subject. To use Piaget’s terms, the concept of plurality is operative, not figurative. It is derived from mental operations, not from sensory material. To establish a plurality, one has to notice the fact that a particular recognition procedure has been carried out and that the same recognition procedure is now being used again in the same experiential context but in a slightly different place. Unless this repetition of subjective operating is taken into account, one cannot distinguish situations where the plural form of a word is appropriate.[7]

Plurality is an elementary part of the knowledge we have to construct ourselves. No postulated external reality can do it for us, and neither can a parent or teacher.

SOME SUGGESTIONS FOR TEACHING

Learning, from the constructivist perspective, is not a stimulus-response phenomenon. It requires self-regulation and the building of conceptual structures through reflection and abstraction. Problems are not solved by the retrieval of rote-learned “right” answers. To solve a problem intelligently, one must first of all see it as one’s own problem. That is to say, one must see it as an obstacle that obstructs one’s progress towards a goal.

The desire to reach what one believes to be at the end of an effort is the most reliable form of motivation. To have searched and found a path to the goal provides incomparably more pleasure and satisfaction than simply to be told that one has given the “right” answer. Having found a viable way of solving a problem does not necessarily eliminate all motivation to search further. At that point, as I mentioned earlier, other criteria may become relevant. The solution found may seem cumbersome, costly, or inelegant, and this may generate the motivation to find another more satisfactory one. In this regard, needless to say, a teacher can be extremely effective in orienting the students’ attention. As Thorndike realized full well, satisfaction is individual and subjective. But the behaviorist dogma that still orients many educational programs obscured this with the assumption that “reinforcement” could be standardized and administered at a trainer’s discretion. The effective motivation to continue learning can be fostered only by leading students to experience the pleasure that is inherent in solving a problem seen and chosen as one’s own.

While the trainer focuses only on the trainee’s performance, the teacher must be concerned with what goes on in the student’s head. The teacher must listen to the student, interpret what the student does and says, and try to build up a “model” of the student’s conceptual structures. This is, of course, a fallible enterprise. But without it, any attempt to change the student’s conceptual structures can be no more than a hit or miss affair.

In the endeavor to arrive at a viable model of the student’s thinking, it is of paramount importance to consider that whatever a student does or says in the context of solving a problem is what, at this moment, makes sense to the student. It may seem to make no sense to the teacher, but unless the teacher can elicit an explanation or generate an hypothesis as to how the student has arrived at the answer, the chances of modifying the student’s conceptual structures are minimal.

7 The appropriate use of a plural obviously requires not only the conceptual construct but also the knowledge of the plural form of the word to be used. A child that has acquired the word “apple” has necessarily heard also the word “apples.” Indeed, the difference between the two words is likely to start the child looking for a perceptual difference. Since this search yields no result on the perceptual level, it may lead the child to focus on its own operating.

In this context, something has to be said on the topic of "misconceptions." With regard to mechanics, for example, students have a considerable range of experience. They have learned to govern the movement of their bodies, they play games with moving objects such as balls, and some of them drive cars at considerable speeds. Inevitably they have derived all sorts of rules from these activities, rules which for the most part are different from those that are considered "correct" in physics. From the physicist's point of view, these notions and rules are misconceptions. But within the students' experiential world, they are quite viable. As long as the counter-examples provided by the teacher are taken from areas that lie outside the students' field of experience, they are unlikely to lead to a change in the students' thinking. Only when students can be led to see as their own a problem in which their approach is manifestly inadequate, will there be any incentive for them to change it. Besides, in teaching science we, too, should learn from our experience and realize that much of what one reads in textbooks as a student will be considered a "misconception" a few decades later. Indeed, it is far more important to teach students to see why a particular conception or theory is considered scientifically viable in a given historical or practical context than to present it as a kind of privileged truth.

Let me close by saying that the best teachers have always known and used all this. But they have known and used it more or less intuitively and often against the official theory of instruction. Constructivism does not claim to have made earth-shaking inventions in the area of education; it merely claims to provide a solid conceptual basis for some of the things that, until now, inspired teachers had to do without theoretical foundation.

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